UPC 126

Fermilab

ON THE INDUCTANCE OF THE DOUBLER MAGNET SYSTEM

R. Shafer

April 2, 1980

Introduction

Nominally the inductance of the Doubler magnet circuit is the total of the inductance of 774 dipoles at approximately 45 mH each, and 216 quadrupoles at 6 mH each, yielding a total inductance of about 36H. However, in considering the operation of the magnet circuit during quench conditions when ac transients may be large and when the currents in the coil bus and the return bus may not be equal, details of all the inductances, including mutual, must be taken into account.

A schematic of the proposed magnet circuit including the return bus protection system is shown in Fig. 1. In order to provide adequate protection to the return bus in the case of a quench, the return bus bypass SCR is gated on, and the 0.1 ohm bypass SCR is gated off, causing the return bus current to drop rapidly to zero. This leaves up to 4000 amps flowing in a 2 km diameter loop in the coil bus, and also linking about 5 km of steel yoke. In order to completely understand how these magnetic volumes enter into consideration, it is necessary to consider separately the self inductances of the return bus and the coil bus.

Return Bus Inductance

In order to estimate the return bus self inductance, we must first recognize that it is approximately a 0.8 cm diameter conductor bent into a 2 km diameter loop. It is surrounded by a steel yoke which we assume to be a toroid roughly 19 cm inside diameter, 28 cm outside diameter, and 5000 m long. We assume the μ of the yoke to be 1000. We divide the magnetic volume into 3 parts:

Region I. Volume inside yoke: i.d. = 0.8 cm, o.d. = 19 cm length = z = 6280 m, μ = 1, $\mu_0 = 4\pi \times 10^{-7}$ H/m.

The energy stored in this volume is

$$E = (1/2)\, LI^2 = (1/2) \int B \cdot H dV = (1/2)(\mu_0\, I^2/2\pi) \int 1/r \; drdz \quad (1)$$

$$\therefore L = \frac{\mu_0 z}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = 4 \text{ mH.} \quad (2)$$

The fact that the conductor is not centered in this volume is only a minor perturbation.

Region II. Volume of yoke: i.d. = 19 cm, o.d. = 28 cm, length = z = 5000 m, μ = 1000.

$$L = \frac{\mu \mu_0 z}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = 390 \text{ mH.} \quad (3)$$

Region III. 2 km loop outside yoke: R = 1 km loop radius, r = 14 cm yoke outside radius, μ = 1

$$L = \mu_0 R \left[\ln\left(\frac{8R}{r}\right) - 2\right] = 11 \text{ mH} \quad (4)$$

Hence the total return bus self inductance is estimated to be roughly 400 mH, a factor of about 30 larger than the estimate in the design report.

Coil Bus Inductance

We also need to make a similar estimate of the coil bus self inductance. We assume all the dipole field energy to be stored inside the yoke in region I. Hence our estimates for the inductances in the 3 regions are:

Region I. (inside yoke) L = 35.7 H

(the total inductance of this region will rise to 36 H when the mutual inductance is taken into account)

Region II. (yoke volume) L = 390 mH

(this includes only the toroidal magnetization, not the dipole, and is assumed to be the same as the return bus)

Region III. (2 km loop outside yoke) L = 11 mH.

Mutual Inductance

In order to complete the circuit, we now need to consider the mutual inductances in these 3 regions.

Region I. The maximum value of this is roughly $(4\text{ mH} \times 36\text{ H})^{1/2}$ = 380 mH. Measurements at B12 indicate that the actual value is roughly 150 mH. This is quite reasonable assuming the coil geometry and the fact that part of the coil actually bucks the major contribution. It is important to note that the sign of this mutual inductance is such that the total stored energy is maximized when the coil and return bus currents are in the normal (opposite) direction.

Region II. In order that there be no stored energy in this region when the coil and return bus currents are normal, this mutual inductance must be - 390 mH. Note sign.

Region III. Similar argument to Region 2. M = -11 mH.

So we can now tabulate the inductances in the circuit

	Self Inductance of Coil	Mutual Inductance	Self Inductance of Return Bus
Region I (inside yoke)	35.7 H	+150 mH (k = +0.4)	4 mH
Region II (yoke)	390 mH	-390 mH (k = -1.0)	390 mH
Region III (2 km loop)	11 mH	-11 mH (k = -1.0)	11 mH
Totals	36.1 H	-251 mH (k = -0.065)	405 mH

where k is defined as $M/(L_1 L_2)^{1/2}$.

Other Coupled Circuits

We now discuss the other circuits which couple to the above magnet circuit. Measurements of the return bus self inductance at B12 failed to show the expected large contribution from the yoke. This was traced to the fact that the cryostat along with external hardware (headers, etc.) formed a conducting loop which linked the yoke. The yoke was in essence a current transformer which was causing current to be induced in the cryostat loop. The most serious loop at the present time seems to be the outer (vacuum) cryostat, which is shorted to the yoke at the ends by virtue of the high clamping forces required to close the yoke gap. The circuit linking the yoke is completed by the steel "L" beams welded along the outside of the yoke. The estimated resistance of the loop is 8 milliohms per magnet, or about 6.7 ohms for the entire ring. Hence this loop has a time constant of

$L/R \cong 60$ millisec. Hence at frequencies above about $R/2\pi L \cong 3$ Hz, the yoke inductance is not visible. Explicitly, the general form of the inductance as a function of frequency is (L_{DC} is the DC inductance, ≈ 390 mH; and $\tau = L_{DC}/R$):

$$L(\omega) = \frac{L_{DC}}{1 + (\omega\tau)^2}. \tag{5}$$

The real part of the impedance however rises with frequency:

$$R(\omega) = R_{DC} + \frac{\omega^2 L_{DC}\,\tau}{1 + (\omega\tau)^2}. \tag{6}$$

These formulas are derived in UPC No. 31.

As the previous measurements on which the quench protection system was designed were made at about 50 Hz, only a negligible fraction of L_{DC} was observable.

The 2 km loop may in addition be shorted by a conducting loop -- i.e., the Main Ring, or beam pipes, or other facilities. The inductance, however, is of little importance relative to the yoke.

Circuit Model

The general magnet circuit then is illustrated in Fig. 2. Dots are used to indicate the polarity of mutual inductance. The SCR states are indicated for normal operation. When a quench occurs, the currents in the coil and return buses are switched thru dump resistors, and the bypass SCR is switched on. The differential equations in this case are

$$L_c \frac{di_c}{dt} + M_{Rc} \frac{di_R}{dt} + i_c R_c = 0$$
$$L_R \frac{di_R}{dt} + M_{Rc} \frac{di_c}{dt} + i_R R_R = 0, \qquad (7)$$

where $L_c = 36.1$ H $R_c = 3$ ohms

$L_R = 405$ mH $R_R = 0.1$ ohms

$M_{Rc} = -251$ mH

i_c = coil bus current

i_R = return bus current

Using $D = d/dt$ we have

$$(L_c D + R_c)\, i_c + M_{Rc}\, D i_R = 0$$
$$(L_R D + R_R)\, i_R + M_{Rc}\, D i_c = 0. \qquad (8)$$

Substituting one equation in the other we get equations of the general form ($\tau_R = L_R/R_R$; $\tau_c = L_c/R_c$; $k^2 = M_{Rc}{}^2/L_R L_c$

$$(1 - k^2)\, D^2 i + \left(\frac{1}{\tau_R} + \frac{1}{\tau_c}\right) Di + \frac{i}{\tau_R \tau_c} = 0. \qquad (9)$$

We assume a solution of the form

$$i = Ae^{+t/\tau}.$$

By direct substitution we find

$$\tau = \frac{-(\tau_R + \tau_c) \pm [(\tau_R - \tau_c)^2 + 4k^2 \tau_R \tau_c]^{1/2}}{2} \quad . \qquad (10)$$

As k is small, the time constants for the discharge of the coil bus and return bus are approximately τ_c and τ_R respectively.

In order to limit the return bus discharge to 7×10^6 amp^2-sec to prevent overheating, $\tau_R \leqslant 0.7$ sec. As the self inductance of the return bus is about 405 mH, this implies a dump resistance of about 0.6 ohms.

Computer Analysis

A computer analysis of the discharge of the circuit with a 0.6 ohm dump resistor, and initial currents of 4500 amps is presented in Figs. 3-6. Note that the maximum current in the cryostat is only about 400 amps. The maximum current in the SCR bypass is about 4000 amps and is reached in about 1 sec. This will induce a field of about (R = 1 km)

$$B = \frac{1}{2}\frac{\mu_0 I}{R} = 25 \text{ milligauss} \tag{11}$$

at the center of the ring, and a field of about

$$B = \frac{\mu_0 I}{2\pi r} = 80 \text{ milligauss} \tag{12}$$

at a distance of r = 100 meters from the magnet.

This flux linking the Main Ring, unless cancelled by counter circulating induced currents in the tunnel structure, would produce a loop voltage of about 45 volts per turn for roughly 1 second on the Main Ring, hence about 2-MeV betatron effect.

Yoke Magnetization

A more serious effect of this net current difference is the toroidal magnetization of the yoke. The B-H magnetization curve for the yoke is given in Fig. 7. The magnetization of the yoke at 4000 amps is ($r = 9.5$ cm)

$$H = \frac{I}{2\pi r} = 7000 \text{ amp-turns/m} = 90 \text{ oersteds} \qquad (13)$$

Hence the yoke will be saturated. Judging from the B•H curve, the yoke will begin saturating when the coil-return bus current difference reaches 500 amps, causing the values of self and mutual inductance in Eqs. (7) to become time dependent. The self inductance of the return bus will drop rapidly to a lower limit of about 15 mH from its initial value of about 400 mH, and the mutual inductance will approach +150 mH from its initial value of -250 mH (note sign change). This non-linear problem can only be solved using numerical techniques.[1]

The severe magnetization in addition will leave some remnant field in the yoke, especially at the thinnest part of the magnetic circuit where the yoke (Wilson) coil is located. As the field in this region is quite small during normal (dipole) excitation, it probably will remain magnetized. What affect this might have on the yoke coil has not been evaluated.

Reference

[1]The non linear properties of the Doubler magnet yoke are estimated in UPC No. 127, Non-Linear Characteristics of the Doubler Magnet Yoke Inductance, R. Shafer, April 8, 1980.

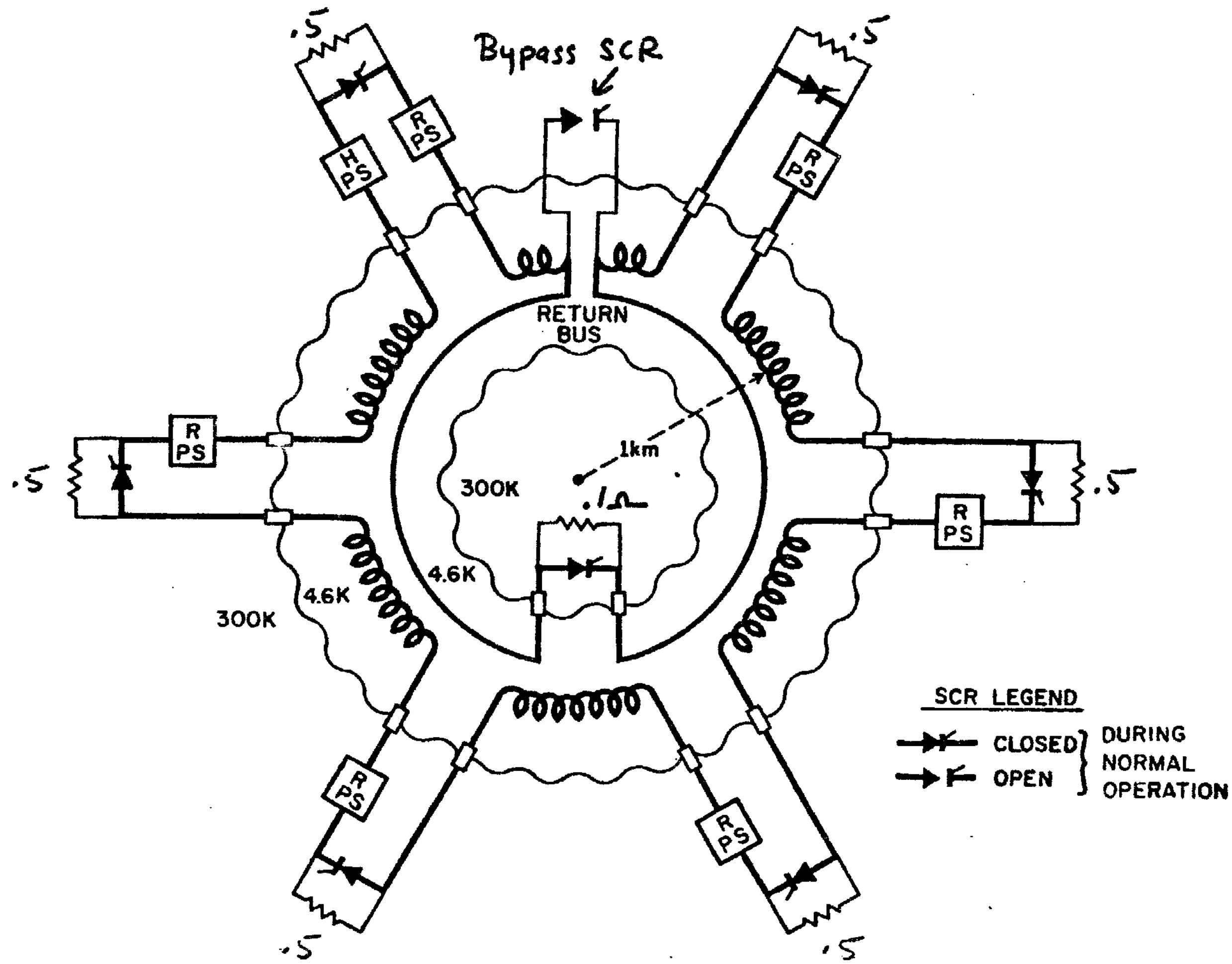


FIGURE 1. The physical layout of the Energy Doubler Magnet circuit showing all the coils on one bus.

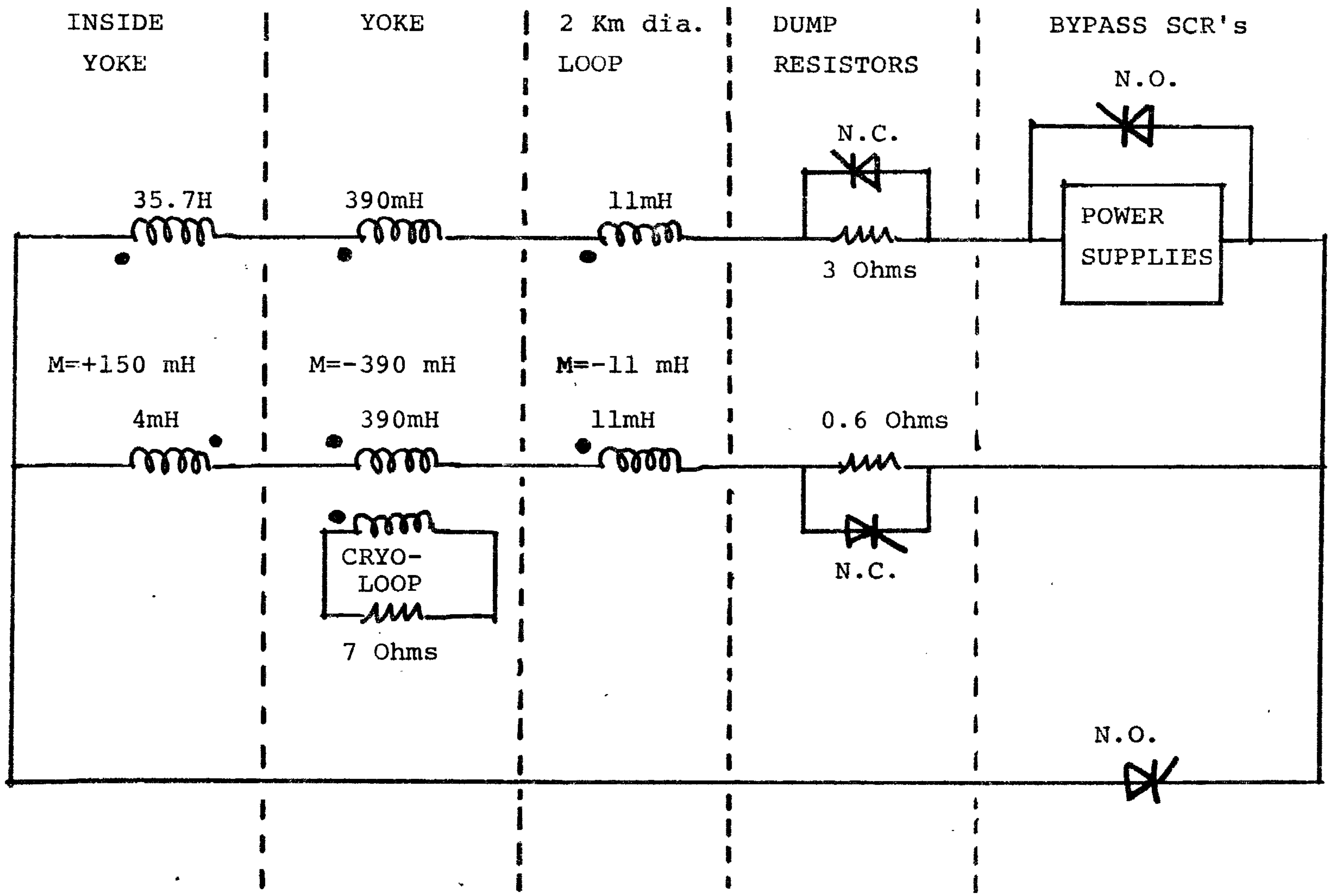


Figure 2. Equivalent circuit for magnet system. Dots show mutual inductance polarity. For SCR's, N.O. and N.C. mean normally open and closed respectively.

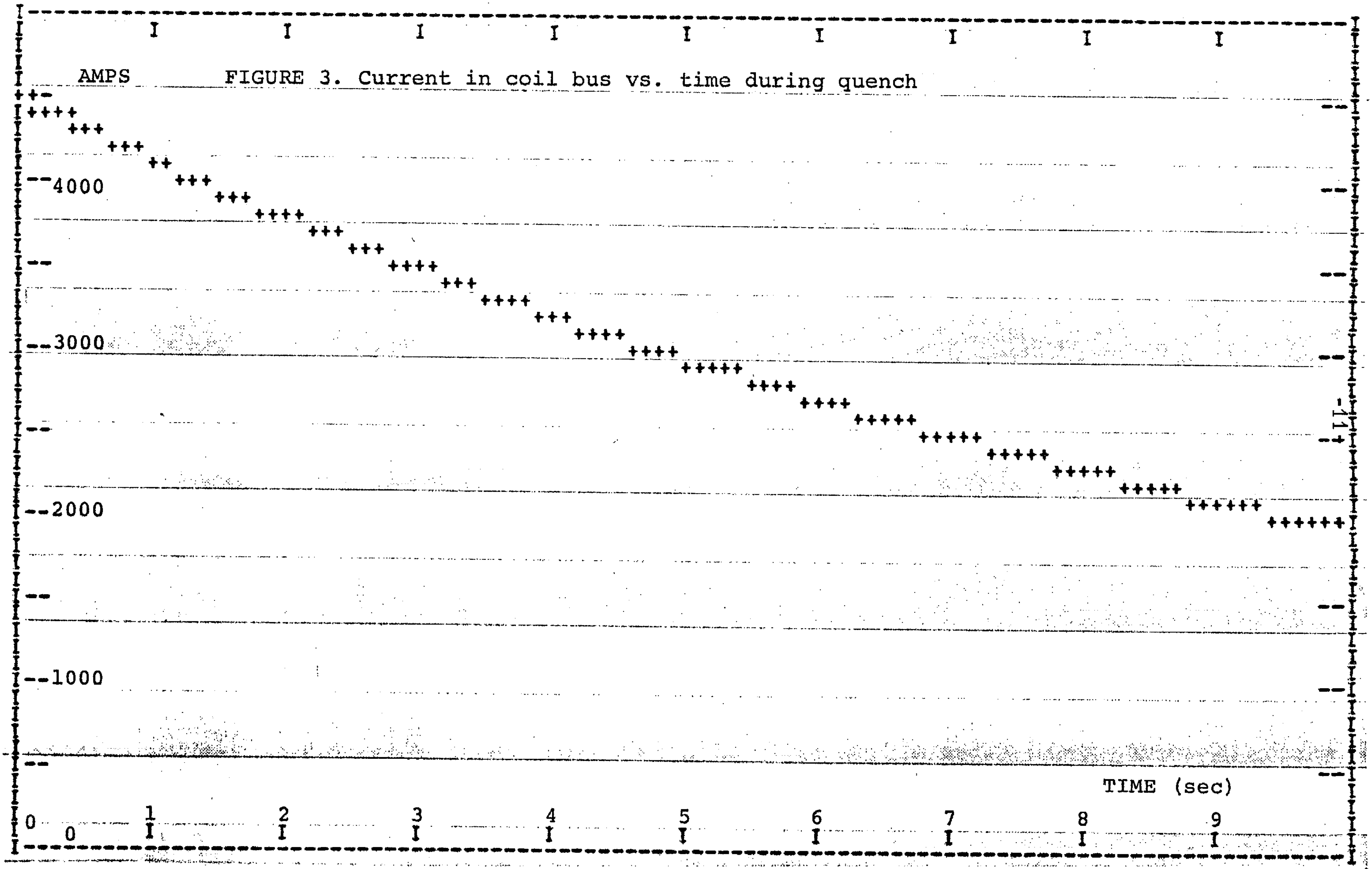


FIGURE 3. Current in coil bus vs. time during quench

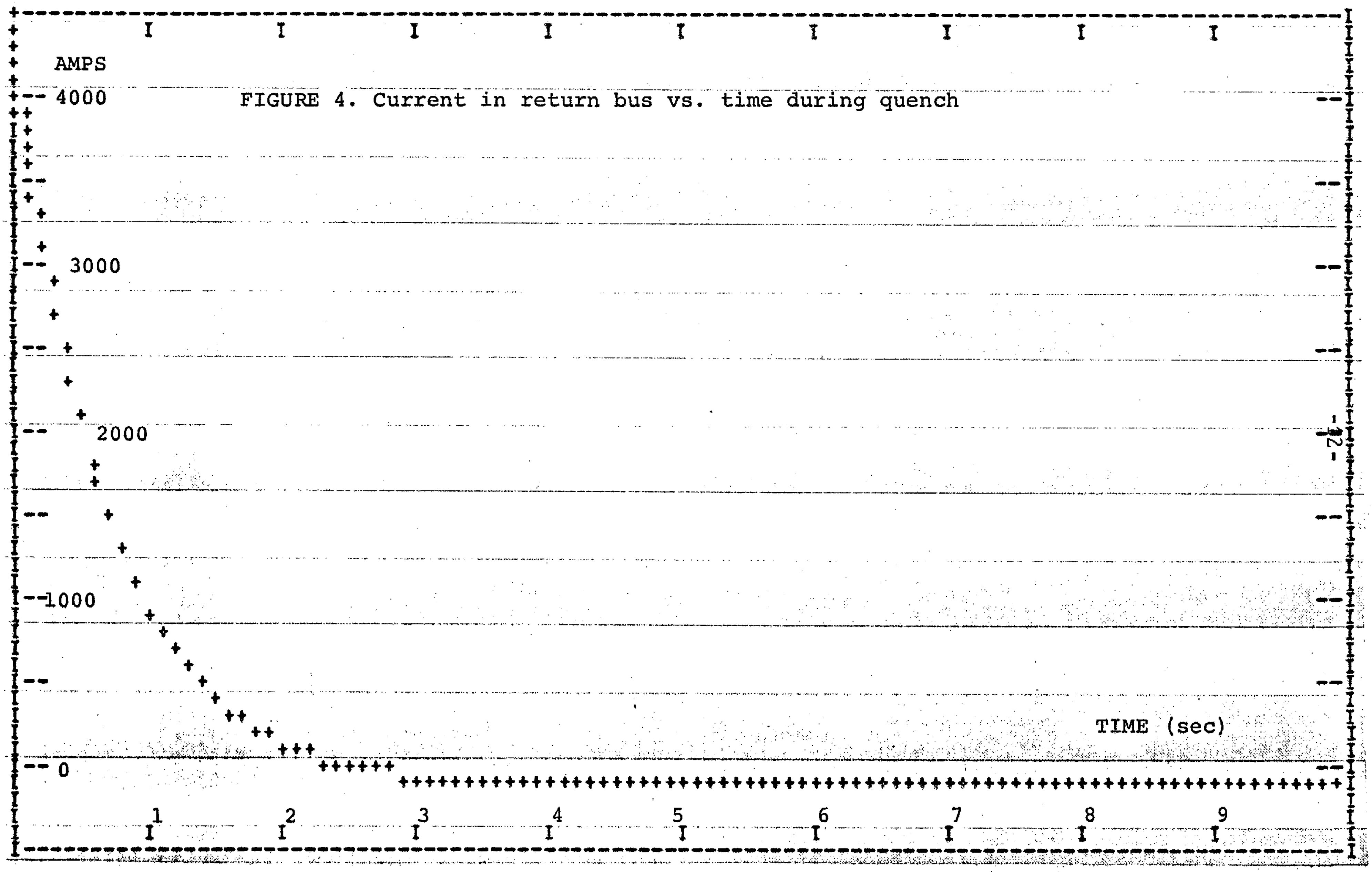
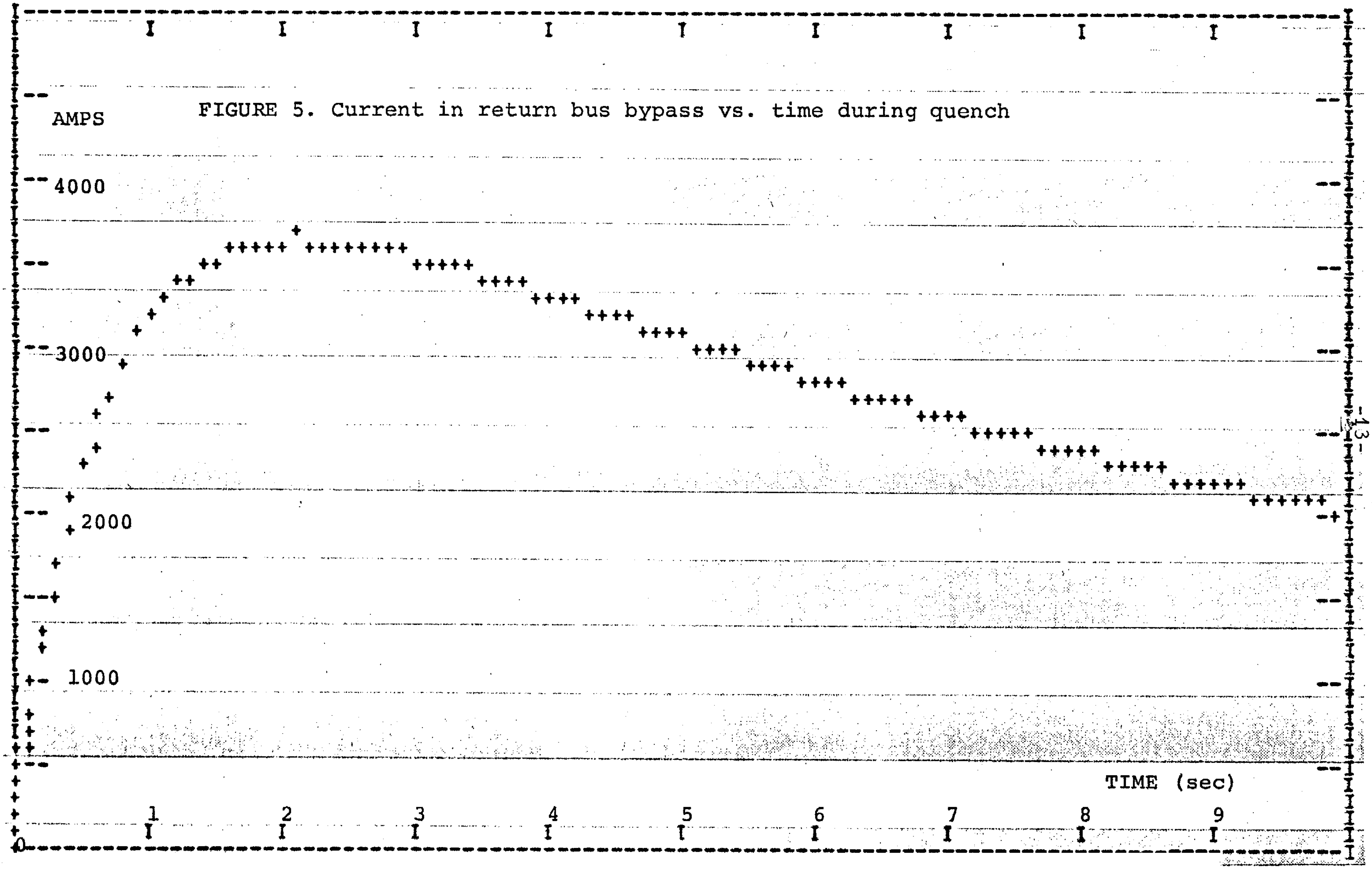


FIGURE 4. Current in return bus vs. time during quench

FIGURE 5. Current in return bus bypass vs. time during quench



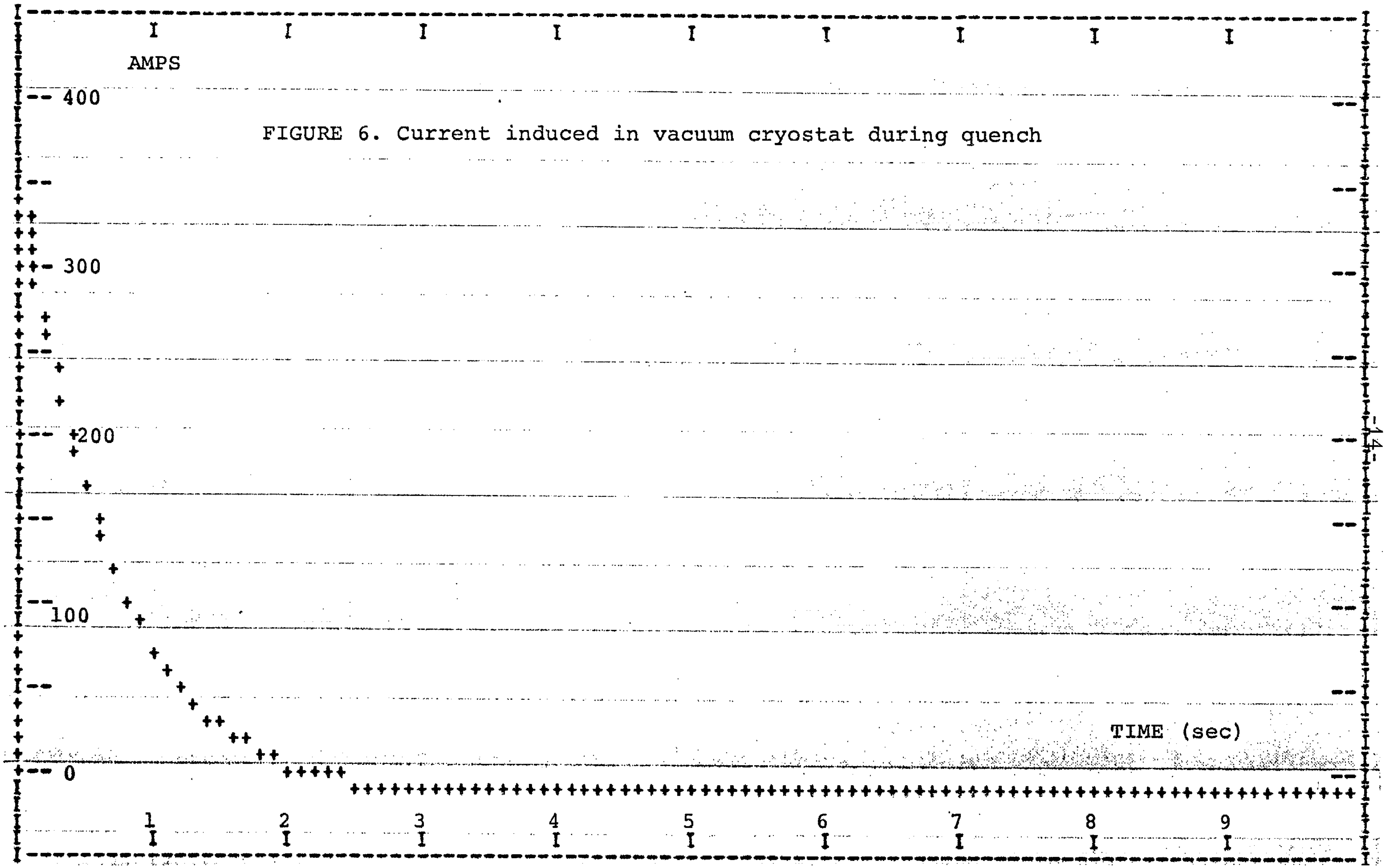


FIGURE 6. Current induced in vacuum cryostat during quench

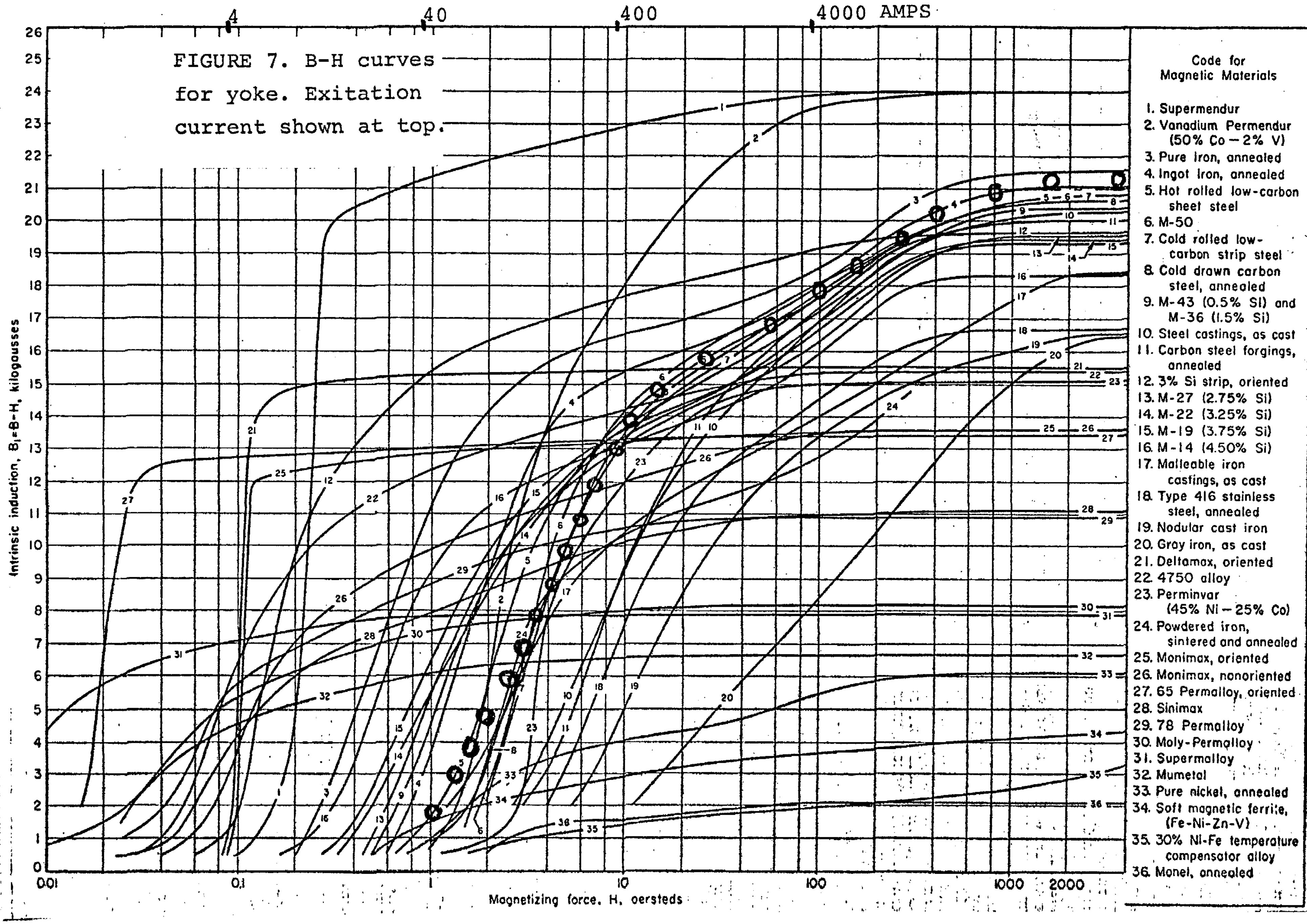


FIGURE 7. B-H curves for yoke. Exitation current shown at top.